بسم الله الرحمن الرحيم

King Abdulaziz University

Review for The Second Exam-Fall 2013

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- أنقر على Start لبداء الاختبار.
- يحتوي هذا الأختبار على ستة وثلاثون سؤالاً.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء الله.

Calculus II
Math202

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Enter Name:

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Start Answer each of the following.

1. $\int \sin^2 x \, dx =$

- $\frac{x}{2} - \frac{\sin(2x)}{4} + C$
- $\frac{x}{2} + \frac{\sin(2x)}{4} + C$
- $\frac{x}{2} - \frac{\cos(2x)}{4} + C$
- $\frac{x}{2} + \frac{\cos(2x)}{4} + C$

2. $\int \cos(5x)\cos(3x)\,dx =$

- ☐ $\frac{1}{4}\sin(2x) - \frac{1}{16}\sin(8x) + C$
- ☐ $\frac{1}{4}\cos(2x) + \frac{1}{16}\cos(8x) + C$
- ☐ $\frac{1}{4}\sin(2x) + \frac{1}{16}\sin(8x) + C$
- ☐ $\frac{1}{4}\cos(2x) - \frac{1}{16}\cos(8x) + C$

3. $\int \tan^3 x \sec^3 x\,dx$

- ☐ $\frac{1}{5}\tan^5 x + \frac{1}{3}\tan^3 x + C$
- ☐ $\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$
- ☐ $\frac{1}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C$
- ☐ $\frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + C$

4. $\int \frac{\sqrt{4-x^2}}{x^2}\,dx =$

- ☐ $\frac{\sqrt{4-x^2}}{x} + \sin^{-1}\left(\frac{x}{2}\right) + C$
- ☐ $-\frac{\sqrt{4-x^2}}{x} + \sin^{-1}\left(\frac{x}{2}\right) + C$
- ☐ $\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\left(\frac{x}{2}\right) + C$
- ☐ $-\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\left(\frac{x}{2}\right) + C$

5. $\int \frac{dx}{\sqrt{x^2-1}} =$

- ☐ $\ln|x - \sqrt{x^2-1}| + C$
- ☐ $\ln|x + \sqrt{x^2-1}| + C$
- ☐ $\sin^{-1} x + C$
- ☐ $\cos^{-1} x + C$

6. $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}} =$

- ☐ $\dfrac{-\sqrt{4-x^2}}{4x}+C$
- ☐ $\dfrac{\sqrt{4-x^2}}{4x}+C$
- ☐ $\dfrac{-\sqrt{4-x^2}}{x}+C$
- ☐ $\dfrac{\sqrt{4-x^2}}{x}+C$

7. $\displaystyle\int \frac{dx}{(x+4)(x+5)} =$

- ☐ $\ln\left|\frac{x-4}{x+5}\right|+C$
- ☐ $-\ln\left|\frac{x+4}{x+5}\right|+C$
- ☐ $\ln\left|\frac{x+5}{x+4}\right|+C$
- ☐ $\ln\left|\frac{x+4}{x+5}\right|+C$

8. $\int \frac{dx}{8-x^2-2x}\,dx =$

- ☐ $\sin^{-1}\left(\frac{x+1}{3}\right)+C$
- ☐ $\sin^{-1}\left(\frac{x+1}{2}\right)+C$
- ☐ $\sin^{-1}\left(\frac{x+1}{8}\right)+C$
- ☐ $\sin^{-1}\left(\frac{x-1}{3}\right)+C$

9. $\int \frac{1}{x^2(x+3)}\,dx =$

- ☐ $\frac{1}{9}\ln\left|\frac{x+3}{x}\right|+\frac{1}{3x}+C$
- ☐ $\frac{-1}{9}\ln\left|\frac{x+3}{x}\right|-\frac{1}{3x}+C$
- ☐ $\frac{1}{9}\ln\left|\frac{x+3}{x}\right|-\frac{1}{3x}+C$
- ☐ $\frac{-1}{9}\ln\left|\frac{x+3}{x}\right|+\frac{1}{3x}+C$

10. $\int \frac{1+\sin x}{\sin^2 x}\, dx =$

- ☐ $-\cot x + \ln|\csc x - \cot x| + C$
- ☐ $\cot x + \ln|\csc x - \cot x| + C$
- ☐ $-\cot x + \ln|\csc x + \cot x| + C$
- ☐ $\cot x + \ln|\csc x + \cot x| + C$

11. $\int \frac{\sqrt{x}}{1+x^3}\, dx =$

- ☐ $\tan^{-1}\left(x^3\right) + C$
- ☐ $\ln\left(|1 + x^3|\right) + C$
- ☐ $\frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right) + C$
- ☐ $\frac{2}{3}\tan^{-1}\left(x^{\frac{3}{2}}\right) + C$

12. $\int \frac{e^{2x}}{1+e^x}\,dx =$

- ☐ $e^x + \ln(1+e^x) + C$
- ☐ $e^x - \ln(1+e^x) + C$
- ☐ $\ln(1+e^x) + C$
- ☐ $\tan^{-1}(e^x) + C$

13. $\int_0^1 x\tan^{-1} x\,dx =$

- ☐ $\frac{\pi+2}{4}$
- ☐ $\frac{\pi-1}{2}$
- ☐ $\frac{\pi-2}{4}$
- ☐ $\frac{\pi+2}{2}$

14. $\int \frac{3x^2 - x + 5}{x(x^2+5)}\, dx =$

- ☐ $\ln|x^3 + 5x| - \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$
- ☐ $\ln|x^3 + 5x| + \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$
- ☐ $\ln|x^3 + 5x| - \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$
- ☐ $\ln|x^3 + 5x| + \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$

15. Use the formula $\int u \tan^{-1} u\, du = \frac{u^2+1}{2}\tan^{-1} u - \frac{u}{2} + C$ to evaluate $\int x^3 \tan^{-1}(x^2)\, dx =$

- ☐ $\frac{x^4+1}{2}\tan^{-1}(x^2) - \frac{x^2}{4} + C.$
- ☐ $\frac{x^4+1}{4}\tan^{-1}(x^2) - \frac{x^2}{4} + C$
- ☐ $\frac{x^4+1}{4}\tan^{-1}(x^2) - \frac{x^2}{2} + C$
- ☐ $\frac{x^4+1}{2}\tan^{-1}(x^2) - \frac{x^2}{2} + C$

16. $\int \frac{1}{x+\sqrt[3]{x}}\,dx =$

- ☐ $\frac{3}{2}\ln\left(x^{\frac{2}{3}}+1\right)+C$
- ☐ $\frac{2}{3}\ln\left(x^{\frac{2}{3}}+1\right)+C$
- ☐ $\frac{3}{2}\ln\left(x^{\frac{3}{2}}+1\right)+C$
- ☐ $\frac{3}{2}\ln\left(x^{\frac{2}{3}}-1\right)+C$

17. $\int_{1}^{e} \frac{1}{x\sqrt[3]{\ln x}}\,dx =$

- ☐ $\frac{3}{2}$
- ☐ $\frac{2}{3}$
- ☐ Divergent
- ☐ $\frac{-3}{2}$

18. $\int_{1}^{\infty} \frac{8}{x^5}\, dx =$

- ☐ 1
- ☐ $\frac{8}{5}$
- ☐ Divergent
- ☐ 2

19. $\int x \sin x \, dx$

- ☐ $x = 4\tan\theta, \quad \frac{-\pi}{2} < \theta < \frac{\pi}{2}.$
- ☐ $x = 4\sin\theta, \quad \frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}.$
- ☐ $x = 4\sec\theta, \quad 0 \leq \theta < \frac{\pi}{2} \text{ or } \pi \leq \theta < \frac{3\pi}{2}$
- ☐ $x = 4\sec\theta\tan\theta, \quad \frac{-\pi}{2} < \theta < \frac{\pi}{2}.$

20. $\int \tan^{-1} x \, dx =$

- ☐ $x\tan^{-1} x - \frac{1}{2}\ln(1 + x^2) + C$
- ☐ $x\tan^{-1} x + \frac{1}{2}\ln(1 + x^2) + C$
- ☐ $x\tan^{-1} x - \ln(1 + x^2) + C$
- ☐ $x\tan^{-1} x + \ln(1 + x^2) + C$

21. $\int \sec^3 x \tan x \, dx =$

- ☐ $\frac{\sec^3 x}{3} + C$
- ☐ $\frac{\tan^3 x}{3} + C$
- ☐ $\frac{\sec^4 x}{4} + C$
- ☐ $\frac{\tan x \sec^2 x}{3} + C$

22. $\int \frac{1}{\sqrt{x^2+4}} \, dx =$

- ☐ $\ln\left|\frac{2}{\sqrt{x^2+4}} - \frac{2}{x}\right| + C$
- ☐ $\ln\left|\frac{2}{\sqrt{x^2+4}} + \frac{2}{x}\right| + C.$
- ☐ $\ln\left|\frac{\sqrt{x^2+4}}{2} - \frac{x}{2}\right| + C.$
- ☐ $\ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| + C.$

23. $\int \frac{x^3+8}{x+2}\, dx =$

- ☐ $\frac{1}{3}x^3 + x^2 + 4x + C$
- ☐ $\frac{1}{3}x^3 - x^2 + 4x + C$
- ☐ $\frac{1}{3}x^3 - x^2 - 4x + C$
- ☐ $\frac{1}{3}x^3 + x^2 - 4x + C$

24. $\int (\cos^4 x - \sin^4 x)\, dx =$

- ☐ $\sin x \cos x + C$
- ☐ $\frac{\sin^5 x}{5} - \frac{\cos^5 x}{5} + C$
- ☐ $x + \frac{1}{2}\cos(2x) + C$
- ☐ $x + \frac{1}{2}\sin(2x) + C$

25. $\int \sqrt{1-\sin(2x)}\,dx =.$

- $\sin x + \cos x + C$
- $\sin x - \cos x + C$
- $-\sin x + \cos x + C$
- $-\sin x - \cos x + C$

26. $\int\limits_{1}^{\infty} 2xe^{-x^2}\,dx =$

- Divergent
- $\frac{1}{e}$
- $\frac{-1}{e^2}$
- $\frac{1}{e^2}$

27. $\int \sin(5)\cos(7x)\,dx =$

- ☐ $\frac{\cos(5)\sin(7x)}{7} + C$
- ☐ $\frac{\sin(5)\sin(7x)}{7} + C$
- ☐ $\frac{\sin(5)\cos(7x)}{7} + C$
- ☐ $\frac{\cos(5)\cos(7x)}{7} + C$

28. $\int \sin^3 x \cos^2 x\,dx =$

- ☐ $-\frac{1}{5}\cos^5 x + \frac{1}{3}\cos^3 x + C$
- ☐ $-\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$
- ☐ $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$
- ☐ $\frac{1}{5}\cos^5 x + \frac{1}{3}\cos^3 x + C$

29. $\int \tan^4 x \sec^2 x \, dx =$

- ☐ $\frac{1}{5}\tan^5 x + \frac{1}{3}\sec^3 x + C$
- ☐ $\tan^5 x + C$
- ☐ $\frac{1}{5}\tan^5 x + C$
- ☐ $\frac{-1}{5}\tan^5 x + C$

30. $\int \cot^3 x \csc^3 x \, dx =$

- ☐ $-\frac{1}{3}\csc^3 x + \frac{1}{5}\csc^5 x + C$
- ☐ $-\frac{1}{3}\csc^3 x - \frac{1}{5}\csc^5 x + C$
- ☐ $\frac{1}{3}\csc^3 x + \frac{1}{5}\csc^5 x + C$
- ☐ $\frac{1}{3}\csc^3 x - \frac{1}{5}\csc^5 x + C$

31. $\int \cos(5x)\cos(4x)\,dx =$

- ☐ $\frac{1}{2}\sin(x) + \frac{1}{18}\sin(9x) + C$
- ☐ $\frac{1}{2}\sin(x) - \frac{1}{18}\sin(9x) + C$
- ☐ $\sin(x) + \frac{1}{9}\sin(9x) + C$
- ☐ $\sin(x) - \frac{1}{9}\sin(9x) + C$

32. $\int \frac{\sqrt{1-x^2}}{x^2}\,dx =$

- ☐ $\frac{\sqrt{1-x^2}}{x} - \sin^{-1}x + C$
- ☐ $\frac{-\sqrt{1-x^2}}{x} - \sin^{-1}x + C$
- ☐ $\frac{-\sqrt{1-x^2}}{x} + \sin^{-1}x + C$
- ☐ $\frac{\sqrt{1-x^2}}{x} + \sin^{-1}x + C$

33. $\int \frac{dx}{\sqrt{x^2+1}} =$

- ☐ $\ln|x - \sqrt{x^2-1}| + C$
- ☐ $\ln|x + \sqrt{x^2+1}| + C$
- ☐ $\sin^{-1} x + C$
- ☐ $\cos^{-1} x + C$

34. $\int \frac{dx}{\sqrt{8-2x-x^2}} =$

- ☐ $\sin^{-1}\left(\frac{x+1}{3}\right) + C$
- ☐ $\sin^{-1}\left(\frac{x-1}{3}\right) + C$
- ☐ $\tan^{-1}\left(\frac{x-1}{3}\right) + C$
- ☐ $\sec^{-1}\left(\frac{x-1}{3}\right) + C$

35. The form of the partial fraction decomposition of $\dfrac{1}{(x+1)^2(x^2+5)} =$

- ☐ $\frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+5}$
- ☐ $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{D}{x^2+5}$
- ☐ $\frac{A}{x+1} + \frac{Bx+C}{(x+1)^2} + \frac{Dx+E}{x^2+5}$
- ☐ $\frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+5}$

36. $\int \frac{x^2}{x+1}\,dx =$

- ☐ $x + 1 + \frac{4x}{x^3-x^2-x+1}$
- ☐ $x - 1 + \frac{4x}{x^3-x^2-x+1}$
- ☐ $x + \frac{4x}{x^3-x^2-x+1}$
- ☐ $-x + 1 + \frac{4x}{x^3-x^2-x+1}$

37. $\int \frac{dx}{(x-4)(x-8)} =$

- [ ] $\frac{-1}{4}\ln|x-8| - \frac{1}{4}\ln|x-4| + C$
- [ ] $\frac{-1}{4}\ln|x-8| + \frac{1}{4}\ln|x-4| + C$
- [ ] $\frac{1}{4}\ln|x-8| + \frac{1}{4}\ln|x-4| + C$
- [ ] $\frac{1}{4}\ln|x-8| - \frac{1}{4}\ln|x-4| + C$

38. $\int \frac{dx}{x^2(x-1)} =$

- [ ] $\frac{1}{x} - \ln|x| + \ln|x-1| + C$
- [ ] $\frac{1}{x} + \ln|x| + \ln|x-1| + C$
- [ ] $\frac{1}{x} - \ln|x| - \ln|x-1| + C$
- [ ] $-\frac{1}{x} - \ln|x| - \ln|x-1| + C$

39. By using the formula $\int \frac{u}{\sqrt{a+bu}}\,du = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}+C$ to evaluate $\int \frac{x}{\sqrt{1+2x}}\,dx =$

- ☐ $\frac{1}{6}(x-1)\sqrt{1+2x}+C.$
- ☐ $\frac{1}{3}(x-1)\sqrt{1+2x}+C$
- ☐ $\frac{1}{3}(2x-1)\sqrt{1+2x}+C$
- ☐ $\frac{1}{3}(x+1)\sqrt{1+2x}+C$

40. $\int e^{\sqrt{x}}\,dx =$

- ☐ $\frac{1}{6}(x-1)\sqrt{1+2x}+C.$
- ☐ $\frac{1}{3}(x-1)\sqrt{1+2x}+C$
- ☐ $\frac{1}{3}(2x-1)\sqrt{1+2x}+C$
- ☐ $\frac{1}{3}(x+1)\sqrt{1+2x}+C$

41. $\int\limits_{2}^{\infty} \frac{1}{x^3}\, dx =$

- ☐ Divergent
- ☐ $\frac{1}{8}$
- ☐ 1
- ☐ $\frac{1}{2}$

42. $\int\limits_{2}^{11} \frac{1}{\sqrt{x-2}}\, dx =$

- ☐ 6
- ☐ 5
- ☐ Divergent
- ☐ 2

43. $\int \frac{1}{x^2+5}\,dx =$

- ☐ $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{5}\right) + C$
- ☐ $\frac{1}{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$
- ☐ $\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$
- ☐ $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$

44. $\int \frac{1}{\sin^2 x + \cos(2x)}\,dx =$

- ☐ $\tan x + C$
- ☐ $\sec x + C$
- ☐ $\cos(2x) + C$
- ☐ $\sin(2x) + C$

45. $\int \frac{9^x+6^x}{3^x}\,dx =$

- ☐ $3^x + 2^x + C$
- ☐ $\frac{9^x}{\ln 9} + \frac{6^x}{\ln 6} + C$
- ☐ $\frac{9^x}{\ln 9} + \frac{2^x}{\ln 2} + C$
- ☐ $\frac{3^x}{\ln 3} + \frac{2^x}{\ln 2} + C$

46. $\int \frac{x}{x^4+2}\,dx =$

- ☐ $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{5}\right) + C$
- ☐ $\frac{1}{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$
- ☐ $\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$
- ☐ $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$

47. $\int \frac{\sin(2x)}{1+\cos^4 x}\,dx =$

- ☐ $-\tan^{-1}(\cos x) + C$
- ☐ $\tan^{-1}\left(\cos^2 x\right) + C$
- ☐ $-\tan^{-1}\left(\cos^2 x\right) + C$
- ☐ $\tan^{-1}(\cos(2x)) + C$

48. $\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx =$

- ☐ $\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$
- ☐ $\frac{2}{3}(x+1)^{\frac{3}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$
- ☐ $(x+1)^{\frac{3}{2}} - x^{\frac{3}{2}} + C$
- ☐ $(x+1)^{\frac{3}{2}} + x^{\frac{3}{2}} + C$

49. $\int \tan^{-1}(\sqrt{x})\,dx =$

- ☐ $x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C$
- ☐ $x\tan^{-1}x + \frac{1}{2}\ln(1+x^2) + C$
- ☐ $x\tan^{-1}x - \ln(1+x^2) + C$
- ☐ $x\tan^{-1}x + \ln(1+x^2) + C$

50. $\int \frac{\sec x \tan x}{\sec^2 x - \sec x}\,dx =$

- ☐ $\ln|x - \sqrt{x^2-1}| + C$
- ☐ $\ln|x + \sqrt{x^2+1}| + C$
- ☐ $\sin^{-1}x + C$
- ☐ $\cos^{-1}x + C$

51. $\int x^5 e^{-x^3}\, dx =$

- ☐ $(x^3+1)e^{-x^3}+C$
- ☐ $-(x^3-1)e^{-x^3}+C$
- ☐ $(x^3-1)e^{-x^3}+C$
- ☐ $-(x^3+1)e^{-x^3}+C$

52. $\int \frac{1}{x\sqrt{4x+1}}\, dx =$

- ☐ $\ln\left|\frac{\sqrt{4x+1}-1}{\sqrt{4x+1}+1}\right|+C$
- ☐ $\ln\left|\frac{\sqrt{4x+1}+1}{\sqrt{4x+1}-1}\right|+C$
- ☐ $2\sqrt{4x+1}+C$
- ☐ $\frac{2}{3}(4x+1)^{3/2}+C$

53. $\int \frac{dx}{x\sqrt{4x^2+1}} =$

- ☐ $\ln\left|\frac{\sqrt{4x^2+1}+1}{2x}\right| + C$
- ☐ $\ln\left|\frac{2x}{\sqrt{4x^2+1}-1}\right| + C$
- ☐ $\ln\left|\frac{\sqrt{4x^2+1}-1}{2x}\right| + C$
- ☐ $\ln\left|\frac{2x}{\sqrt{4x^2+1}+1}\right| + C$

54. $\int \frac{dx}{x(x^4+1)} =$

- ☐ $\ln\left(\sqrt{\frac{x^4+1}{x^4}}\right) + C$
- ☐ $\ln\left(\sqrt{\frac{x^4+1}{x^4}}\right) + C$
- ☐ $\ln\left(\sqrt[4]{\frac{x^4+1}{x^4}}\right) + C$
- ☐ $\ln\left(\sqrt[4]{\frac{x^4}{x^4+1}}\right) + C$

55. $\int \sqrt{1+e^x}\, dx =$

- ☐ $2\sqrt{1+e^x} + \ln\left|\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right| + C$
- ☐ $\sqrt{1+e^x} + \ln\left|\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right| + C$
- ☐ $2\sqrt{1+e^x} + \ln\left|\frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}\right| + C$
- ☐ $\sqrt{1+e^x} + \ln\left|\frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}\right| + C$

56. $\int \frac{xe^x}{\sqrt{1+e^x}}\, dx =$

- ☐ $2\sqrt{1+e^x} + \ln\left|\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right| + C$
- ☐ $\sqrt{1+e^x} + \ln\left|\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right| + C$
- ☐ $2\sqrt{1+e^x} + \ln\left|\frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}\right| + C$
- ☐ $\sqrt{1+e^x} + \ln\left|\frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1}\right| + C$

57. $\int \frac{1}{1+e^x}\,dx =$

- ☐ $\sec^{-1}(e^x) + C$
- ☐ $\tan^{-1}(e^x) + C$
- ☐ $\ln\left(\frac{e^x}{1+e^x}\right) + C$
- ☐ $\ln\left(\frac{1+e^x}{e^x}\right) + C$

58. $\int \frac{e^x}{2e^{2x}+e^x+1}\,dx =$

- ☐ $\sec^{-1}(e^x) + C$
- ☐ $\tan^{-1}(e^x) + C$
- ☐ $\ln\left(\frac{e^x}{1+e^x}\right) + C$
- ☐ $\ln\left(\frac{1+e^x}{e^x}\right) + C$

59. $\int \frac{x}{\sqrt{x^2+2x+5}}\,dx =$

- ☐ $\sqrt{x^2+2x+5} - \ln\left|\frac{x+1+\sqrt{x^2+2x+5}}{2}\right| + C$
- ☐ $\sqrt{x^2+2x+5} + \ln\left|\frac{x+1+\sqrt{x^2+2x+5}}{2}\right| + C$
- ☐ $\sqrt{x^2+2x+5} - \ln\left|\frac{x-1+\sqrt{x^2+2x+5}}{2}\right| + C$
- ☐ $\sqrt{x^2+2x+5} + \ln\left|\frac{x-1+\sqrt{x^2+2x+5}}{2}\right| + C$

60. $\int \frac{\sqrt{1+\sqrt{x}}}{x}\,dx =$

- ☐ $2\sqrt{1+\sqrt{x}} + 2\ln\left(\frac{\sqrt{1+\sqrt{x}}-1}{\sqrt{1+\sqrt{x}}+1}\right) + C$
- ☐ $4\sqrt{1+\sqrt{x}} + 2\ln\left(\frac{\sqrt{1+\sqrt{x}}+1}{\sqrt{1+\sqrt{x}}-1}\right) + C$
- ☐ $4\sqrt{1+\sqrt{x}} + 2\ln\left(\frac{\sqrt{1+\sqrt{x}}-1}{\sqrt{1+\sqrt{x}}+1}\right) + C$
- ☐ $\sqrt{1+\sqrt{x}} + 2\ln\left(\frac{\sqrt{1+\sqrt{x}}-1}{\sqrt{1+\sqrt{x}}+1}\right) + C$

End

Score: Correct

Answers:

Points: Percent:

Letter Grade:

Solutions to Quizzes

Solution to 1.

$$\begin{aligned}\int \sin^2 x\,dx &= \int \frac{1-\cos(2x)}{2}\,dx \qquad \text{use } \sin^2 x = \frac{1-\cos(2x)}{2}\\ &= \frac{1}{2}\int [1-\cos(2x)]\,dx \quad \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C\\ &= \frac{1}{2}[x - \frac{1}{2}\sin(2x)] + C\\ &= \frac{x}{2} - \frac{\sin(2x)}{4} + C.\end{aligned}$$

■

Solution to 2.

$$\int \cos(5x)\cos(3x)\,dx$$

$$= \int \frac{\cos(5x-3x)+\cos(5x+3x)}{2}\,dx \quad \text{use } \cos A\cos B = \frac{\cos(A-B)+\cos(A+B)}{2}.$$

$$= \frac{1}{2}\int [\cos(2x)+\cos(8x)]\,dx \quad \text{use } \int \cos(ax)\,dx = \frac{1}{a}\sin(ax)+C.$$

$$= \frac{1}{2}[\frac{1}{2}\sin(2x)+\frac{1}{8}\sin(8x)]+C$$

$$= \frac{1}{4}\sin(2x)+\frac{1}{16}\sin(8x)+C.$$

■

Solution to 3.

$$\int \tan^3 x \sec^3 x \, dx$$

$$= \int \tan^2 x \sec^2 x \ \sec x \tan x \, dx \qquad \text{use } \tan^2 x = \sec^2 x - 1.$$

$$= \int [\sec^2 x - 1] \sec^2 x \ \sec x \tan x \, dx \quad \text{use } u = \sec x \quad du = \sec x \tan x \, dx.$$

$$= \int [u^2 - 1] u^2 \, du$$

$$= \int [u^4 - u^2] \, du$$

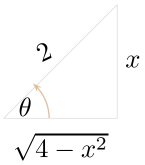
$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C.$$

■

Solution to 4.

Since we have $\sqrt{4-x^2}$ then



$$x = 2\sin\theta$$
$$dx = 2\cos\theta\, d\theta$$
$$x^2 = (2\sin\theta)^2 = 4\sin^2\theta \sqrt{9-x^2} = 2\cos\theta.$$

$$\begin{aligned}
\int \frac{\sqrt{4-x^2}}{x^2}\,dx &= \int \frac{2\cos\theta}{4\sin^2\theta}\,2\cos\theta\,d\theta \\
&= \int \frac{4\cos^2\theta}{4\sin^2\theta}\,d\theta && \text{use } \cot\theta = \frac{\cos\theta}{\sin\theta}. \\
&= \int \cot^2\theta\,d\theta && \text{use } \cot^2\theta = \csc^2\theta - 1 \\
&= \int [\csc^2\theta - 1]\,d\theta && \text{use } \int \csc^2\theta\,d\theta = -\cot\theta \\
&= -\cot\theta - \theta + C && \text{from the triangle } \cot\theta = \frac{\sqrt{4-x^2}}{x} \\
&= -\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\left(\frac{x}{2}\right) + C.
\end{aligned}$$

■

Solution to 5.

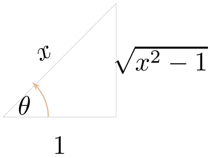
$$\int \frac{dx}{\sqrt{x^2-1}} = \int \frac{1}{\sqrt{x^2-1^2}}\,dx \qquad \text{use } \int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right| + C.$$

$$= \ln|x + \sqrt{x^2-1} + C.$$

Another solution: Let $x = \sec\theta$,

$dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-1} = \tan\theta.$

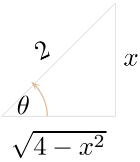


$$\begin{aligned}\int \frac{dx}{\sqrt{x^2-1}} &= \int \frac{1}{\tan\theta}\,\sec\theta\tan\theta\,d\theta \\ &= \int \sec\theta\,d\theta \\ &= \ln|\sec\theta+\tan\theta| + C \\ &= \ln|x+\sqrt{x^2-1}| + C.\end{aligned}$$

■

Solution to 6. Let

$$
\begin{aligned}
x &= 2\sin\theta, \\
dx &= 2\cos\theta\, d\theta \\
\sqrt{4-x^2} &= 2\cos\theta \\
x^2 &= 4\sin^2\theta.
\end{aligned}
$$



$$
\begin{aligned}
\int \frac{dx}{x^2\sqrt{4-x^2}} &= \int \frac{2\cos\theta\, d\theta}{4\sin^2\theta 2\cos\theta} \\
&= \int \frac{d\theta}{4\sin^2\theta} \\
&= \frac{1}{4}\int \csc^2\theta\, d\theta \\
&= \frac{-\cot\theta}{4} + C \\
&= \frac{-\sqrt{4-x^2}}{4x} + C.
\end{aligned}
$$

■

Solution to 7.

$$\frac{1}{(x+4)(x+5)} = \frac{A}{x+4} + \frac{B}{x+5} \qquad \text{Multiply both sides by } (x+4)(x+5).$$

$$1 = A(x+5) + B(x+4)$$

$$\text{if } x = -4 : 1 = A \Rightarrow A = 1$$

$$\text{if } x = -5 : 1 = -B \Rightarrow B = -1$$

Hence $A = 1, B = -1$.

$$\begin{aligned}\int \frac{dx}{(x+4)(x+5)} &= \int \left[\frac{1}{x+4} + \frac{-1}{x+5}\right] dx \\ &= \int \frac{1}{x+4}\,dx - \int \frac{1}{x+5}\,dx \\ &= \ln|x+4| - \ln|x+5| + C \\ &= \ln\left|\frac{x+4}{x+5}\right| + C.\end{aligned}$$

■

Solution to 8.

$$\begin{aligned} 8 - x^2 - 2x &= 8 - [x^2 + 2x], \\ &= 8 - [x^2 + 2x + 1^2 - 1^2], \\ &= 8 - (x^2 + 2x + 1) - (-1), \\ &= 9 - (x+1)^2 = 3^2 - (x+1)^2. \end{aligned}$$

$$\begin{aligned} \int \frac{dx}{\sqrt{8 - x^2 - 2x}} &= \int \frac{1}{\sqrt{3^2 - (x+1)^2}}\, dx \\ &= \sin^{-1}\left(\frac{x+1}{3}\right) + C. \end{aligned}$$

■

Solution to 9.

$$\frac{1}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$$

Multiply both sides by $x^2(x+3)$.

$$1 = Ax(x+3) + B(x+3) + Cx^2$$

$$\text{if } x = 0 : 1 = 3B \Rightarrow B = \frac{1}{3}$$

$$\text{if } x = -3 : 1 = 9C \Rightarrow C = \frac{1}{9}$$

$$\text{if } x = 1 : 1 = 4A + 4B + C \Rightarrow 4A = 1 - 4B - C$$

$$\Rightarrow 4A = 1 - \frac{4}{3} - \frac{1}{9}$$

$$\Rightarrow 4A = \frac{9 - 12 - 1}{3} = \frac{-4}{9}$$

$$\Rightarrow A = \frac{-4}{9}.\frac{1}{4} = \frac{-1}{9}$$

Hence $A = \frac{-1}{9}, B = \frac{1}{3}, C = \frac{1}{9}$

$$\begin{aligned}
\int \frac{dx}{x^2(x+3)} &= \int \left[\frac{\frac{-1}{9}}{x} + \frac{\frac{1}{3}}{x^2} + \frac{\frac{1}{9}}{x+3} \right] dx \\
&= \frac{-1}{9} \int \frac{1}{x}\, dx + \frac{1}{3} \int x^{-2}\, dx + \frac{1}{9} \int \frac{1}{x+3}\, dx \\
&= \frac{-1}{9} \ln|x| + \frac{1}{9} \ln|x+3| + \frac{1}{3} \frac{x^{-1}}{-1} + C \\
&= \frac{1}{9} \ln \left| \frac{x+3}{x} \right| - \frac{1}{3x} + C.
\end{aligned}$$

■

Solution to 10.

$$\int \frac{1+\sin x}{\sin^2 x}\,dx$$

$$= \int [\frac{1}{\sin^2 x} + \frac{\sin x}{\sin^2 x}]\,dx \qquad \text{use } \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}.$$

$$= \int [\csc^2 x + \csc x]\,dx \qquad \text{use } \int \csc^2 x\,dx = -\cot x + C \quad \int \csc x\,dx = \ln|\csc x - \cot x| + C$$

$$= -\cot x + \ln|\csc x - \cot x| + C.$$

■

Solution to 11.

$$
\begin{aligned}
w &= \sqrt{x} \\
w^2 &= x \\
x^3 &= (w^2)^3 = w^6 \\
dx &= 2w\,dw
\end{aligned}
$$

$$\begin{aligned}
\int \frac{\sqrt{x}}{1+x^3}\,dx &= \int \frac{w}{1+w^6}\,2w\,dw \\
&= \int \frac{2w^2}{1+w^6}\,dw \\
&= 2\int \frac{w^2}{1+(w^3)^2}\,dw \quad \text{use } \int \frac{f'(x)}{a^2+[f(x)]^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right)+C. \\
&= \frac{1}{3}.2\int \frac{3w^2}{1+[w^3]^2}\,dw \\
&= \frac{2}{3}\tan^{-1}\left(w^3\right)+C \quad \text{use } w^3 = (\sqrt{x})^3 = x^{\frac{3}{2}} \\
&= \frac{2}{3}\tan^{-1}\left(x^{\frac{3}{2}}\right)+C.
\end{aligned}$$

■

Solution to 12.

$$\begin{aligned}
w &= e^x \\
1+w &= 1+e^x \\
dw &= e^x\,dx \\
\int \frac{e^{2x}}{1+e^x}\,dx &= \int \frac{e^x}{1+e^x}\,e^x\,dx \\
&= \int \frac{w}{1+w}\;dw \\
&= \int \frac{w+1-1}{w+1}\,dw \qquad \text{use } w = w+1-1. \\
&= \int [\frac{w+1}{w+1} + \frac{-1}{w+1}]\,dw \\
&= \int [1 - \frac{1}{w+1}]\,dw \\
&= w - \ln|w+1| + C \qquad w = e^x \\
&= e^x - \ln(1+e^x) + C.
\end{aligned}$$

Solution to 13. Let

$$u = \tan^{-1} x \qquad dv = x\,dx$$

$$du = \frac{1}{1+x^2}\,dx \qquad v = \frac{1}{2}x^2$$

Therefore,

$$\int_0^1 \tan^{-1} x \, dx$$

$$= \frac{1}{2}x^2 \tan^{-1} x \Big|_0^1 - \int_0^1 \frac{\frac{1}{2}x^2}{x^2+1} \, dx$$

$$= \left[\frac{1}{2}(1)^2 \tan^{-1} 1 - \frac{1}{2}(0)^2 \tan^{-1} 0\right] - \frac{1}{2} \int_0^1 \frac{x^2+1-1}{x^2+1} \, dx \qquad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$= \frac{\pi}{8} - \frac{1}{2} \int_0^1 [1 - \frac{1}{x^2+1}] \, dx$$

$$= \frac{\pi}{8} - \frac{1}{2} \left[x - \tan^{-1} x\right]\Big|_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2}[1 - \tan^{-1} -(0 - \tan^{-1} 0)]$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} = \frac{\pi}{4} - \frac{1}{2} = \frac{\pi - 2}{4}.$$

■

Solution to 14.

$$\frac{3x^2 - x + 5}{x(x^2+5)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 5}$$

Multiply both sides by $x(x^2+5)$.

$$\begin{aligned}
3x^2 - x + 5 &= A(x^2+5) + (Bx + C)x \\
3x^2 - x + 5 &= (A+B)x^2 + Cx + 5A \\
\text{if } x = 0 : 5 &= 5A \Rightarrow A = 1 \\
\text{coff } x^2 : 3 &= A + B \Rightarrow B = 3 - A \\
&\Rightarrow B = 3 - 1 = 2 \\
\text{coff } x : -1 &= C \Rightarrow C = -1
\end{aligned}$$

Hence $A = 1, B = 2, C = -1$

$$\begin{aligned}\int \frac{dx}{x(x^2+5)} &= \int \left[\frac{1}{x} + \frac{2x-1}{x^2+5}\right] dx \\ &= \int \left[\frac{1}{x} + \frac{2x}{x^2+5} - \frac{1}{x^2+5}\right] dx \\ &= \ln|x| + \ln\left(x^2+5\right) - \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C \\ &= \ln\left|x^3+5x\right| - \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C.\end{aligned}$$

■

Solution to 15.

$$\begin{aligned}
u &= x^2 \\
du &= 2x\,dx \\
\frac{du}{2} &= x\,dx \\
\int x^3 \tan^{-1}(x^2)\,dx &= \int x^2 \tan^{-1}(x^2)\,x\,dx \\
&= \int u \tan^{-1} u\ \frac{1}{2}du \\
&= \frac{1}{2}\int u \tan^{-1} u\,du && \text{use the given formula} \\
&= \frac{1}{2}[\frac{u^2+1}{2}\tan^{-1} u - \frac{u}{2}] + C \\
&= \frac{u^2+1}{4}\tan^{-1} u - \frac{u}{4} + C && \text{use } u = x^2 \\
&= \frac{x^4+1}{4}\tan^{-1}(x^2) - \frac{x^2}{4} + C
\end{aligned}$$

■

Solution to 16.

$$\begin{aligned} w &= \sqrt[3]{x} \\ w^3 &= x \\ 3w^2 dw &= dx \\ \int \frac{1}{x+\sqrt[3]{x}}\,dx &= \int \frac{1}{w^3+w}\,3w^2\,dw \\ &= \int \frac{3w^2}{w(w^2+1)}\,dw \\ &= 3\int \frac{w}{w^2+1}\,dw \\ &= 3\frac{1}{2}\int \frac{2w}{w^2+1}\,dw \\ &= \frac{3}{2}\ln\left(w^2+1\right)+C \quad \text{use } w=\sqrt[3]{x} \\ &= \frac{3}{2}\ln\left(x^{\frac{2}{3}}+1\right)+C \end{aligned}$$

■

Solution to 17. Since $\frac{1}{x\sqrt[3]{\ln x}}$ is discontinuous at $1 \in [1, e]$ we have improper integral.

$$\begin{aligned}
\int_1^e \frac{1}{x\sqrt[3]{\ln x}}\,dx &= \lim_{t\to 1^+} \int_t^e (\ln x)^{-1/3}\,\frac{1}{x}\,dx \\
&= \lim_{t\to 1^+} \left[\frac{3}{2}(\ln x)^{2/3}\right]_t^e \\
&= \lim_{t\to 1^+} \left[\frac{3}{2}(\ln e)^{2/3} - \frac{3}{2}(\ln t)^{2/3}\right] \\
&= \frac{3}{2}.
\end{aligned}$$

■

Solution to 18.

$$\begin{aligned}\int_1^\infty \frac{8}{x^5}\,dx &= \lim_{t\to\infty}\int_1^t \frac{8}{x^5}\,dx \\ &= \lim_{t\to\infty}\int 8x^{-5}\,dx \quad \text{Use } \int [f(x)]^n\, f'(x)dx = \frac{[f(x)]^{n+1}}{n+1}+C. \\ &= \lim_{t\to\infty}\left[\frac{-8}{4}x^{-4}\right]_1^t \\ &= \lim_{t\to\infty}\left[\frac{-2}{x^4}\right]_1^t \\ &= \lim_{t\to\infty}\left[\frac{-2}{t^4}+2\right] \\ &= 2.\end{aligned}$$

Hence $\int_1^\infty \frac{8}{x^5}\,dx = 2$ converge. ■

Solution to 19.

Alternate signs	u and its derivatives	dv and its antiderivatives
$+$	x	$\sin x$
$-$	1	$-\cos x$
$+$	0	$-\sin x$
		$\cos x$

Differentiate until you get 0.

Hence $\int x \sin x \, dx = -x\cos x - \sin x + C$. ■

Solution to 20.

Let

$$u = \tan^{-1} x \qquad dv = dx$$
$$du = \frac{1}{1+x^2}\,dx \qquad v = x$$

Therefore,

$$\begin{aligned}
&\int \tan^{-1} x\,dx \\
&= x\tan^{-1} x - \int \frac{x}{1+x^2}\,dx \\
&= x\tan^{-1} x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx \\
&= x\tan^{-1} x - \frac{1}{2}\ln\left(1+x^2\right) + C.
\end{aligned}$$

■

Solution to 21.

$$\begin{aligned}
\int \sec^3 x \tan x \, dx &= \int \sec^2 x \, \sec x \tan x \, dx \\
&= \int \sec^2 x \, \sec x \tan x \, dx \quad \text{let } u = \sec x, \; du = \sec x \tan x \, dx. \\
&= \int u^2 \, du \\
&= \frac{u^3}{3} + C \\
&= \frac{\sec^3 x}{3} + C
\end{aligned}$$

■

Solution to 22.

$$\int \frac{dx}{\sqrt{x^2+4}} = \int \frac{1}{\sqrt{x^2+2^2}}\,dx \qquad \text{use } \int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left|\frac{x+\sqrt{x^2+a^2}}{a}\right| + C.$$

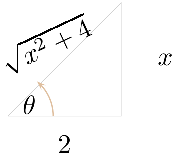
$$= \ln\left|\frac{x+\sqrt{x^2+4}}{2}\right| + C$$

$$= \ln\left|\frac{x}{2} + \frac{\sqrt{x^2+4}}{2}\right| + C.$$

Another solution: Let $x = 2\tan\theta$,

$dx = \sec^2\theta\, d\theta$

$\sqrt{x^2+4} = 2\sec\theta.$



$$\begin{aligned}
\int \frac{dx}{\sqrt{x^2+4}} &= \int \frac{1}{2\sec\theta}\, 2\sec^2\theta\, d\theta \\
&= \int \sec\theta\, d\theta \\
&= \ln|\sec\theta + \tan\theta| + C \\
&= \ln|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}| + C.
\end{aligned}$$

■

Solution to 23. Since $x^3+8 = x^3+2^3 = (x+2)(x^2-2x+4)$ then $\frac{x^3+8}{x+2} = \frac{(x+2)(x^2-2x+4)}{x+2} = x^2-2x+4.$

$$\int \frac{x^3+8}{x+2}\,dx = \int (x^2-2x+4)\,dx = \frac{1}{3}x^3 - x^2 + 4x + C$$

■

Solution to 24.

$$\int (\cos^4 x - \sin^4 x)\, dx$$

$$= \int (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)\, dx \quad \text{use } a^4 - b^4 = (a^2 - b^2)(a^2 + b^2).$$

$$= \int \cos(2x)\, dx \quad \text{use } \cos^2 x - \sin^2 x = \cos(2x),\ \cos^2 x + \sin^2 x = 1.$$

$$= \frac{1}{2}\sin(2x) + C \quad \text{use } \int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + C.$$

$$= \frac{1}{2} 2 \sin x \cos x + C \quad \text{use } \sin(2x) = 2 \sin x \cos x.$$

$$= \sin x \cos x + C.$$

■

Solution to 25. Remember that $1 = \cos^2 x + \sin^x$, $\sin(2x) = 2\cos x \sin x$ and $a^2 - 2ab + b^2 = (a-b)^2$. Also $\sqrt{a^2} = |a| = \begin{cases} a, & \text{if } a \geq 0; \\ -a, & \text{if } a < 0. \end{cases}$ so we may sometime use $\sqrt{a^2} = a$.

$$\begin{aligned}
1 - \sin(2x) &= \cos^2 x + \sin^x - 2\cos x \sin x \\
&= \cos^2 x - 2\cos x \sin x + \sin^2 x \\
&= (\cos x - \sin x)^2 \\
\int \sqrt{1 - \sin(2x)}\, dx &= \int \sqrt{(\cos x - \sin x)^2}\, dx \\
&= \int (\cos x - \sin x)\, dx \\
&= \sin x + \cos x + C.
\end{aligned}$$

■

Solution to 26.

$$\int_1^\infty 2xe^{-x^2}\,dx = \lim_{t\to\infty}\int_1^t 2xe^{-x^2}\,dx$$

$$= -\lim_{t\to\infty}\int_1^t e^{-x^2}\,-2x\,dx \quad \text{Use } \int e^{f(x)}f'(x)dx = e^{f(x)} + C.$$

$$= -\lim_{t\to\infty}\left[e^{-x^2}\right]_1^t$$

$$= -\lim_{t\to\infty}\left[\frac{1}{e^{t^2}} - \frac{1}{e^2}\right]$$

$$= -[0 - \frac{1}{e^2}] = \frac{1}{e^2}.$$

Hence $\int_1^\infty 2xe^{-x^2}\,dx = \frac{1}{e^2}$ converge. ■

Solution to 27.

$$\int \sin(5)\cos(7x)\,dx = \sin(5)\int \cos(7x)\,dx \quad \text{Use } \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + C.$$
$$= \frac{\sin(5)\sin(7x)}{7} + C.$$

■

Solution to 28.

$$\int \sin^3 x \cos^2 x \, dx = \int \sin^2 x \cos^2 x \, \sin x \, dx \qquad \text{use } \sin^2 x = 1 - \cos^2 x$$

$$= \int (1 - \cos^2 x) \cos^2 x \, \sin x \, dx \qquad \text{Let } u = \cos x,$$

$$du = -\sin x \, dx \Rightarrow -du = \sin x \, dx$$

$$= \int (1 - u^2) u^2 \, (-du)$$

$$= \int (u^4 - u^2) \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C \qquad \text{replace } u = \cos x.$$

$$= \frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C.$$

■

Solution to 29.

$$\begin{aligned}\int \tan^4 x \sec^2 x \, dx &= \int \tan^4 x \; \sec^2 x \, dx \quad \text{Let } u = \tan x, \; du = \sec^2 x \, dx \\ &= \int u^4 \, du \\ &= \frac{u^5}{5} + C \qquad \text{replace } u = \tan x. \\ &= \frac{1}{5} \tan^5 x + C.\end{aligned}$$

■

Solution to 30.

$$\begin{aligned}\int \cot^3 x \csc^3 x\,dx &= \int \cot^2 x \csc^2 x\ \csc x \cot x\,dx && \text{use } \cot^2 x = \csc^2 x - 1\\ &= \int (\csc^2 x - 1)\csc^2 x\ \csc x \cot x\,dx && \text{Let } u = \csc x,\end{aligned}$$

$du = -\csc x \cot x\,dx \Rightarrow -du = \csc x \cot x\,dx$

$$\begin{aligned} &= \int (u^2-1)u^2\,(-du)\\ &= \int (u^2 - u^4)\,du\\ &= \frac{u^3}{3} - \frac{u^5}{5} + C && \text{replace } u = \csc x.\\ &= \frac{1}{3}\csc^3 x - \frac{1}{5}\csc^5 x + C.\end{aligned}$$

■

Solution to 31.

$$\int \cos(5x)\cos(4x)\,dx$$

$$= \int \frac{\cos(5x-4x)+\cos(5x+4x)}{2}\,dx \quad \text{use } \cos A \cos B = \frac{\cos(A-B)+\cos(A+B)}{2}.$$

$$= \frac{1}{2}\int [\cos(x)+\cos(9x)]\,dx \quad \text{use } \int \cos(ax)\,dx = \frac{1}{a}\sin(ax)+C.$$

$$= \frac{1}{2}[\sin(x)+\frac{1}{9}\sin(9x)]+C$$

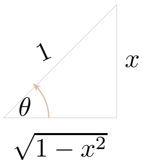
$$= \frac{1}{2}\sin(x)+\frac{1}{18}\sin(9x)+C.$$

■

Solution to 32.

Since we have $\sqrt{1-x^2}$ then

$$x = \sin\theta$$
$$dx = \cos\theta\, d\theta$$
$$x^2 = (\sin\theta)^2 = \sin^2\theta$$
$$\sqrt{1-x^2} = \cos\theta.$$



$$\begin{aligned}
\int \frac{\sqrt{1-x^2}}{x^2}\,dx &= \int \frac{\cos\theta}{\sin^2\theta}\,\cos\theta\,d\theta \\
&= \int \frac{\cos^2\theta}{\sin^2\theta}\,d\theta && \text{use } \frac{\cos y}{\sin y} = \cot y. \\
&= \int \cot^2\theta\,d\theta && \text{use } \cot^2\theta = \csc^2\theta - 1. \\
&= \int [\csc^2\theta - 1]\,d\theta \\
&= -\cot\theta - \theta + C \\
&= \frac{-\sqrt{1-x^2}}{x} - \sin^{-1}x + C.
\end{aligned}$$

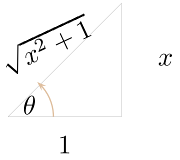
■

Solution to 33.

$$\int \frac{dx}{\sqrt{x^2-1}} = \int \frac{1}{\sqrt{x^2-1^2}}\,dx \qquad \text{use } \int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left|\frac{x+\sqrt{x^2+a^2}}{a}\right| + C.$$

$$= \ln\left|x+\sqrt{x^2-1} + C.\right.$$

Another solution: Let $x = \tan\theta$,
$dx = \sec^2\theta\, d\theta$
$\sqrt{x^2+1} = \sec\theta.$



$$\begin{aligned}\int \frac{dx}{\sqrt{x^2+1}} &= \int \frac{1}{\sec\theta}\,\sec^2\theta\, d\theta \\ &= \int \sec\theta\, d\theta \\ &= \ln|\sec\theta + \tan\theta| + C \\ &= \ln|x + \sqrt{x^2+1}| + C.\end{aligned}$$

■

Solution to 34.

$$\begin{aligned} 8-2x-x^2 &= 8-[x^2+2x], \\ &= 8-(x^2+2x+1^2-1^2), \\ &= 8-(x^2-2x+1-1), \\ &= 8-(x-1)^2+1 \\ &= 9-(x-1)^2. \end{aligned}$$

$$\begin{aligned} \int \frac{dx}{\sqrt{8-2x-x^2}} &= \int \frac{1}{\sqrt{9-(x-1)^2}}\,dx \text{use } \int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}\,dx = \sin^{-1}\left(\frac{f(x)}{a}\right)+C. \\ &= \sin^{-1}\left(\frac{x-1}{3}\right)+C. \end{aligned}$$

■

Solution to 35. Since $(x+1)^2(x^2+5)$, then we have two repeated linear factors and one irreducible factor hence

$$\frac{1}{(x+1)^2(x^2+5)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+5}$$

■

Solution to 36.

Using long division we have

$$
\begin{array}{r|l}
 & x-1 \\
\hline
x+1 & x^2 \\
 & -x^2 + {}^{-}x \\
\hline
 & -x \\
 & +-x - {}^{+}1 \\
\hline
 & 1
\end{array}
$$

Hence $\frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}$.

Thus $\int \frac{x^2}{x+1}\, dx = \int [x - 1 + \frac{1}{x+1}]\, dx = \frac{1}{2}x^2 - x + \ln|x+1| + C$ ■

Solution to 37.

$$\frac{1}{(x-4)(x-8)} = \frac{A}{x-4} + \frac{B}{x-8}$$

Multiply both sides by $(x-4)(x-8)$.

$$1 = A(x-8) + B(x-4)$$

$$\text{if } x = 4 : 1 = -4A \Rightarrow A = \frac{-1}{4}$$

$$\text{if } x = 8 : 1 = 4B \Rightarrow B = \frac{1}{4}$$

Hence $A = \frac{-1}{4}, B = \frac{1}{4}$.

$$\begin{aligned}\int \frac{dx}{(x-4)(x-8)} &= \int \left[\frac{\frac{-1}{4}}{x-4} + \frac{\frac{1}{4}}{x-8}\right] dx \\ &= \frac{-1}{4}\int \frac{1}{x-4}\,dx + \frac{1}{4}\int \frac{1}{x-8}\,dx \\ &= \frac{-1}{4}\ln|x-4| + \frac{1}{4}\ln|x-8| + C \\ &= \frac{1}{4}\ln|x-8| - \frac{1}{4}\ln|x-4| + C \\ &= \frac{1}{4}\ln\left|\frac{x-8}{x-4}\right| + C.\end{aligned}$$

■

Solution to 38. Since $x^2(x-1)$, then we have three linear factors one of them is repeated. Hence

$$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

Multiply both sides by $x^2(x-1)$.

$$1 = Ax(x-1) + B(x-1) + Cx^2$$

$$\text{if } x = 0 : 1 = -B \Rightarrow B = -1$$

$$\text{if } x = 1 : 1 = C \Rightarrow C = 1$$

$$\text{if } x = 2 : 1 = 2A + B + 4C$$

$$\Rightarrow 2A = 1 - B - 4C$$

$$2A = 1 - (-1) - 4(1)$$

$$\Rightarrow 2A = -2$$

$$\Rightarrow A = -1$$

Hence $A = -1, B = -1, C = 1$.

$$\begin{aligned}\int \frac{dx}{x^2(x-1)} &= \int \left[\frac{-1}{x} + \frac{-1}{x^2} + \frac{1}{x-1}\right] dx \\ &= -\int \frac{1}{x}\, dx - \int x^{-2}\, dx + \int \frac{1}{x-1}\, dx \\ &= -\ln|x| - [\frac{x^{-1}}{-1} + \ln|x-1| + C \\ &= \frac{1}{x} - \ln|x| + \ln|x-1| + C \\ &= \frac{1}{x} + \ln\left|\frac{x-1}{x}\right| + C.\end{aligned}$$

■

Solution to 39.

$$
\begin{aligned}
u &= x \\
du &= dx \\
a &= 1 \\
b &= 2 \\
&\int \frac{x}{\sqrt{1+2x}}\,dx \qquad \text{use the given formula} \\
&= \frac{2}{3(2)^2}(2x-2(1))\sqrt{1+2x}+C \\
&= \frac{2}{12}(2x-2)\sqrt{1+2x}+C \\
&= \frac{4}{12}(x-1)\sqrt{1+2x}+C \\
&= \frac{1}{3}(x-1)\sqrt{1+2x}+C
\end{aligned}
$$

■

Solution to 40.

$$\begin{aligned} w &= \sqrt{x} \\ w^2 &= x \\ 2w\,dw &= dx \\ &\int e^{\sqrt{x}}\,dx \\ &= \int e^{w} 2w\,dw \\ &= \int 2we^{w}\,dw \end{aligned}$$

Alternate signs	u and its derivatives	dv and its antiderivatives
$+$	$2w$	e^w
$-$	2	e^w
$+$	0	e^w
		e^w

Differentiate until you get 0.

$\int 2we^w \, dw = 2we^w - 2e^w + C$

$$
\begin{aligned}
w &= \sqrt{x} \\
w^2 &= x \\
2w\,dw &= dx \\
&\int e^{\sqrt{x}}\,dx \\
&= \int e^w 2w\,dw \\
&= \int 2we^w\,dw \\
&= 2we^w - 2e^w + C \\
&= 2(w-1)e^w + C \qquad \text{use } w = \sqrt{x} \\
&= 2(\sqrt{x}-1)e^{\sqrt{x}} + C
\end{aligned}
$$

■

Solution to 41.

$$\begin{aligned}
\int_2^\infty \frac{1}{x^3}\,dx &= \lim_{t\to\infty}\int_2^t x^{-3}\,dx \\
&= \lim_{t\to\infty}\left[\frac{x^{-2}}{-2}\right]_2^t \\
&= \lim_{t\to\infty}\left[\frac{-1}{2x^2}\right]_2^t \\
&= \lim_{t\to\infty}\left[\frac{-1}{2t^2}-\frac{-1}{8}\right] \\
&= [0-\frac{-1}{8}] = \frac{1}{8}.
\end{aligned}$$

Hence $\int_1^\infty 2xe^{-x^2}\,dx = \frac{1}{e^2}$ converge. ■

Solution to 42. Since $\frac{1}{\sqrt{x-2}}$ is discontinuous at $2 \in [2, 11]$ we have improper integral.

$$\begin{aligned}
\int_2^{11} \frac{1}{\sqrt{x-2}}\,dx &= \lim_{t\to 2^+} \int_t^{11} (x-2)^{-1/2}\,dx \\
&= \lim_{t\to 2^+} \left[2(x-2)^{1/2}\right]_t^{11} \\
&= \lim_{t\to 2^+} \left[2\sqrt{x-2}\right]_t^{11} \\
&= \lim_{t\to 2^+} \left[2\sqrt{9} - 2\sqrt{t-2}\right] \\
&= 6.
\end{aligned}$$

■

Solution to 43.

$$\int \frac{1}{x^2+5}\,dx = \int \frac{1}{x^2+5}\,dx \qquad \text{use } \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

$$= \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+C$$

■

Solution to 44.

$$\int \frac{1}{\sin^2 x + \cos(2x)}\,dx$$

$$= \int \frac{1}{\sin^2 x + \cos^2 x - \sin^2 x}\,dx \quad \text{use } \cos(2A) = \cos^2 A - \sin^2 A$$

$$= \int \frac{1}{\cos^2 x}\,dx \quad \text{use } \frac{1}{\cos x} = \sec x$$

$$= \int \sec^2 x\,dx$$

$$= \tan x + C.$$

■

Solution to 45.

$$\int \frac{9^x + 6^x}{3^x}\,dx$$

$$= \int \left[\frac{9^x}{3^x} + \frac{6^x}{3^x}\right] dx \qquad \text{use } \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$= \int \left[\left(\frac{9}{3}\right)^x + \left(\frac{6}{3}\right)^x\right] dx \qquad \text{use } \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$= \int [3^x + 2^x]\,dx$$

$$= \frac{3^x}{\ln 3} + \frac{2^x}{\ln 2} + C.$$

■

Solution to 46.

$$\begin{aligned}\int \frac{x}{x^4+2}\,dx &= \int \frac{x}{(x^2)^2+2}\,dx\\ &= \frac{1}{2}\int \frac{2x}{(x^2)^2+2}\,dx \qquad \text{use } \int \frac{f'(x)}{[f(x)]^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right)+C\\ &= \frac{1}{2}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2}{\sqrt{2}}\right)+C\\ &= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2}{\sqrt{2}}\right)+C\end{aligned}$$

■

Solution to 47.

$$\int \frac{\sin(2x)}{1+\cos^4 x}\,dx = \int \frac{2\cos x \sin x}{1+(\cos^2)^2}\,dx \qquad \text{use } \sin(2x) = 2\cos x \sin x \text{ , } \cos^4 x = (\cos^2 x)^2$$

$$= -\int \frac{-2\cos x \sin x}{1+(\cos^2)^2}\,dx \qquad \text{use } (\cos^2 x)' = -2\cos x \sin x$$

$$= -\tan^{-1}\left(\cos^2 x\right) + C \qquad \text{use } \int \frac{f'(x)}{[f(x)]^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right) + C$$

■

Solution to 48.

$$\begin{aligned}\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx &= \int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,\frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}\,dx \quad \text{use } (a-b)(a+b)=a^2-b^2\\ &= \int \frac{\sqrt{x+1}+\sqrt{x}}{(x+1)-x}\,dx\\ &= \int [(x+1)^{\frac{1}{2}} - x^{\frac{1}{2}}]\,dx \quad \text{use } \int [f(x)]^n\,dx = \frac{[f(x)]^{n+1}}{n+1} + C\\ &= \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C\end{aligned}$$

■

Solution to 49.

Let

$$\begin{aligned} w &= \sqrt{x} \\ w^2 &= x \\ 2w\,dw &= dx \\ \int \tan^{-1}\left(\sqrt{x}\right)dx &= \int \tan^{-1} w\, 2w\, dw \\ &= \int 2w \tan^{-1} w\, dw. \end{aligned}$$

Let

$$u = \tan^{-1} w \qquad dv = 2w\,dw$$

$$du = \frac{1}{1+w^2}\,dx \qquad v = w^2$$

Therefore,

$$
\begin{aligned}
&\int \tan^{-1}\left(\sqrt{x}\right) dx \\
&= \int 2w \tan^{-1} w \, dw \\
&= w^2 \tan^{-1} w - \int \frac{w^2}{1+w^2} \, dw \\
&= w^2 \tan^{-1} w - \int \frac{w^2+1-1}{1+w^2} \, dw \\
&= w^2 \tan^{-1} w - \int [1 - \frac{1}{1+w^2}] \, dw \\
&= w^2 \tan^{-1} w - w + \tan^{-1} w + C \\
&= x \tan^{-1}\left(\sqrt{x}\right) + \tan^{-1}\left(\sqrt{x}\right) - \sqrt{x} + C \\
&= (x+1) \tan^{-1}\left(\sqrt{x}\right) - \sqrt{x} + C.
\end{aligned}
$$

■

Solution to 50.

$$\int \frac{\sec x \tan x}{\sec^2 x - \sec x}\,dx = \int \frac{\sec x \tan x}{\sec x(\sec x - 1)}\,dx \quad \text{use } \sec^2 x - \sec x = \sec x(\sec x - 1)$$

$$= \int \frac{\tan x}{\sec x - 1}\frac{\cos x}{\cos x}\,dx \quad \text{use } \tan x \cos x = \sin x,\ \cos x \sec x = 1$$

$$= \int \frac{\sin x}{1 - \cos x}\,dx \quad \text{use } (1 - \cos x)' = \sin x$$

$$= \ln|1 - \cos x| + C \quad \text{use } \int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$= \ln\left|1 - \cos x\frac{\sec x}{\sec x}\right| + C$$

$$= \ln\left|\frac{\sec x - 1}{\sec x}\right| + C$$

Another solution: Let $w = \sec x$,
$dx = \sec x \tan x\,dx$

$$\int \frac{\sec x \tan x}{\sec^2 x - \sec x}\, dx = \int \frac{1}{\sec^2 x - \sec x}\, \sec x \tan x\, dx$$
$$= \int \frac{1}{w^2 - w}\, dw$$
$$= \int \frac{1}{w(w-1)}\, dw$$

$$\frac{1}{w(w-1)} = \frac{A}{w} + \frac{B}{w-1}$$ Multiply both sides by $w(w-1)$.

$$1 = A(w-1) + Bw$$
$$\text{if } w = 0 : 1 = -A \Rightarrow A = -1$$
$$\text{if } x = 1 : 1 = B \Rightarrow B = 1$$

Hence $A = -1, B = 1$.

$$\begin{aligned}
\int \frac{\sec x \tan x}{\sec^2 x - \sec x}\, dx &= \int \frac{1}{\sec^2 x - \sec x} \sec x \tan x \, dx \\
&= \int \frac{1}{w^2 - w}\, dw \\
&= \int \frac{1}{w(w-1)}\, dw \\
&= \int \left[\frac{-1}{w} + \frac{1}{w-1} \right] dw \\
&= \int \left[\frac{1}{w-1} - \frac{1}{w} \right] dw \\
&= \ln|w-1| - \ln|w| + C \\
&= \ln\left| \frac{w-1}{w} \right| + C \\
&= \ln\left| \frac{\sec x - 1}{\sec x} \right| + C
\end{aligned}$$

■

Solution to 51.

$$
\begin{aligned}
w &= x^3 \\
dw &= 3x^2\,dx \\
\int x^5 e^{-x^3}\,dx &= \int x^3 e^{-x^3}\,x^2\,dx \\
&= \frac{1}{3}\int x^3 e^{-x^3}\,3\,x^2\,dx_{w\,=\,x^3} \\
&= \frac{1}{3}\int w e^{-w}\,dw
\end{aligned}
$$

Alternate signs	u and its derivatives	dv and its antiderivatives
$+$	w	e^{-w}
$-$	1	$-e^{-w}$
$+$	0	e^{-w}
		$-e^{-w}$

Differentiate until you get 0.

Hence

$$\begin{aligned}\int x^5 e^{-x^3}\,dx &= \int x^3 e^{-x^3}\,x^2\,dx \\ &= \frac{1}{3}\int x^3 e^{-x^3}\,3\,x^2\,dx \qquad w = x^3 \\ &= \frac{1}{3}\int w e^{-w}\,dw \\ &= -we^{-w} - e^{-w} + C \\ &= -x^3 e^{-x^3} - e^{-x^3} + C \\ &= -(x^3+1)e^{-x^3} + C\end{aligned}$$

■

Solution to 52.

$$\begin{aligned} w &= \sqrt{4x+1} \\ w^2 &= 4x+1 \\ w^2-1 &= 4x \\ \frac{w^2-1}{4} &= x \\ 2w\,dw &= 4\,dx \\ \frac{w}{2}\,dw = dx & \\ \int \frac{1}{x\sqrt{4x+1}}\,dx &= \int \frac{1}{\frac{w^2-1}{4}w}\,\frac{w}{2}\,dw \\ &= \frac{4}{2}\int \frac{1}{w^2-1}\,dw \qquad \int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C \\ &= 2\frac{1}{2}\ln\left|\frac{w-a}{w+a}\right| + C \\ &= \ln\left|\frac{\sqrt{4x+1}-1}{\sqrt{4x+1}+1}\right| + C \end{aligned}$$

■

Solution to 53.

$$
\begin{aligned}
w &= \sqrt{4x^2+1} \\
w^2 &= 4x^2+1 \\
w^2-1 &= 4x^2 \\
\frac{w^2-1}{4} &= x^2 \\
\frac{2}{4}w\,dw &= 2x\,dx
\end{aligned}
$$

$$\frac{w}{4}\,dw = x\,dx$$

$$\int \frac{1}{x\sqrt{4x^2+1}}\,dx = \int \frac{1}{x\sqrt{4x^2+1}}\frac{x}{x}\,dx = \int \frac{1}{x^2\sqrt{4x^2+1}}\,x\,dx$$

$$= \int \frac{1}{\frac{w^2-1}{4}w}\,\frac{w}{4}\,dw$$

$$= \frac{4}{4}\int \frac{1}{w^2-1}\,dw \qquad \int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{w-a}{w+a}\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{\sqrt{4x^2+1}-1}{\sqrt{4x^2+1}+1}\,\frac{\sqrt{4x^2+1}-1}{\sqrt{4x^2+1}-1}\right| + C$$

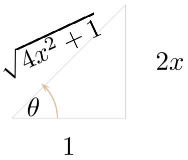
$$= \frac{1}{2}\ln\left|\frac{(\sqrt{4x^2+1}-1)^2}{4x^2}\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{(\sqrt{4x^2+1}-1)^2}{(2x)^2}\right| + C = \frac{1}{2}\ln\left|\frac{\sqrt{4x^2+1}-1}{2x}\right|^2 + C$$

$$= 2.\frac{1}{2}\ln\left|\frac{\sqrt{4x^2+1}-1}{2x}\right| + C = \ln\left|\frac{\sqrt{4x^2+1}-1}{2x}\right| + C$$

Another solution: $\int \frac{dx}{x\sqrt{4x^2+1}} = \int \frac{dx}{x\sqrt{(2x)^2+1}}\,dx$

Let $2x = \tan\theta$,
$x = \frac{1}{2}\tan\theta$,
$2dx = \sec^2\theta\,d\theta$
$dx = \frac{1}{2}\sec^2\theta\,d\theta$
$\sqrt{4x^2+1} = \sec\theta.$



$$\begin{aligned}
\int \frac{dx}{x\sqrt{4x^2+1}} &= \int \frac{1}{\frac{1}{2}\tan\theta\sec\theta}\,\frac{1}{2}\sec^2\theta\,d\theta \\
&= \int \frac{\sec\theta}{\tan\theta}\,d\theta \qquad \frac{\sec\theta}{\tan\theta} = \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \csc\theta \\
&= \int \csc\theta\,d\theta \\
&= \ln|\csc\theta - \cot\theta| + C \\
&= \ln\left|\frac{\sqrt{4x^2+1}}{2x} - \frac{1}{2x}\right| + C \\
&= \ln\left|\frac{\sqrt{4x^2+1}-1}{2x}\right| + C.
\end{aligned}$$

■

Solution to 54.

$$w = x^4$$
$$dw = 4x^3\,dx$$

$$\int \frac{dx}{x(x^4+1)} = \int \frac{1}{x(x^4+1)}\frac{x^3}{x^3}\,dx = \int \frac{1}{x^4(x^4+1)}\,x^3\,dx$$
$$= \frac{1}{4}\int \frac{1}{x^4(x^4+1)}\,4x^3\,dx \quad \text{use } w = x^4$$
$$= \frac{1}{4}\int \frac{1}{w(w+1)}\,dw \quad \text{use partial fraction for } \frac{1}{w(w+1)}$$

$$\frac{1}{w(w+1)} = \frac{A}{w} + \frac{B}{w+1} \quad \text{Multiply both sides by } w(w+1).$$
$$1 = A(w+1) + Bw$$
$$\text{if } w = 0 : 1 = A \Rightarrow A = 1$$
$$\text{if } w = -1 : 1 = -B \Rightarrow B = -1$$

Hence $A = 1, B = -1$. Then

$$\begin{aligned}
\int \frac{dx}{x(x^4+1)} &= \int \frac{1}{x(x^4+1)} \frac{x^3}{x^3}\, dx = \int \frac{1}{x^4(x^4+1)}\, x^3\, dx \\
&= \frac{1}{4} \int \frac{1}{x^4(x^4+1)}\, 4x^3\, dx \qquad \text{use } w = x^4 \\
&= \frac{1}{4} \int \frac{1}{w(w+1)}\, dw \qquad \text{use partial fraction for } \frac{1}{w(w+1)} \\
&= \frac{1}{4} \int \left[\frac{1}{w} - \frac{1}{w+1} \right] dw \\
&= \frac{1}{4} [\ln|w| - \ln|w+1|] + C \\
&= \frac{1}{4} \ln\left| \frac{w}{w+1} \right| + C \\
&= \frac{1}{4} \ln\left(\frac{x^4}{x^4+1} \right) + C \qquad \text{use } w = x^4 \text{ and } x^4 > 0 \\
&= \ln\left(\sqrt[4]{\frac{x^4}{x^4+1}} \right) + C
\end{aligned}$$

■

Solution to 55.

$$\begin{aligned} w &= \sqrt{1+e^x} \\ w^2 &= 1+e^x \\ w^2-1 &= e^x \\ \ln\left(w^2-1\right) &= \ln e^x = x \\ \frac{2w}{w^2-1}\,dw &= dx \end{aligned}$$

$$\begin{aligned}
\int \sqrt{1+e^x}\,dx &= \int w\frac{2w}{w^2-1}\,dw \\
&= \int \frac{2w^2}{w^2-1}\,dw \\
&= 2\int \frac{w^2}{w^2-1}\,dw \qquad w^2 = w^2-1+1 \\
&= 2\int \frac{w^2-1+1}{w^2-1}\,dw \qquad \frac{a+b}{c} = \frac{a}{c}+\frac{b}{c} \\
&= 2\int \left[\frac{w^2-1}{w^2-1}+\frac{1}{w^2-1}\right]\,dw \\
&= 2\int \left[1+\frac{1}{w^2-1}\right]\,dx \qquad \int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C \\
&= 2\left[w+\frac{1}{2}\ln\left|\frac{w-1}{w+1}\right|\right]+C = 2w+\ln\left|\frac{w-1}{w+1}\right|+C \\
&= 2\sqrt{1+e^x}+\ln\left|\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right|+C
\end{aligned}$$

■

Solution to 56.

$$
\begin{aligned}
w &= \sqrt{1+e^x} \\
w^2 &= 1+e^x \\
w^2-1 &= e^x \\
\ln\left(w^2-1\right) &= \ln e^x = x \\
2w\,dw &= e^x\,dx \\
\int \frac{xe^x}{\sqrt{1+e^x}}\,dx &= \int \frac{x}{\sqrt{1+e^x}}\,e^x\,dx \\
&= \int \frac{\ln\left(w^2-1\right)}{w}\,2w\,dw \\
&= \int 2\ln\left(w^2-1\right)dw
\end{aligned}
$$

$$\int \frac{xe^x}{\sqrt{1+e^x}}\,dx = \int 2\ln\left(w^2-1\right)dw \quad \text{Integration by part } u = \ln\left(w^2-1\right)\; dv = 2\,dw$$

$$\text{Integration by part } du = \frac{2w}{w^2-1}\,dw \; v = 2w$$

$$= 2w\ln\left(w^2-1\right) - \int \frac{4w^2}{w^2-1}\,dw$$

$$= 2w\ln\left(w^2-1\right) - 4\int \frac{w^2-1+1}{w^2-1}\,dw \quad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$= 2w\ln\left(w^2-1\right) - 4\int \left[1 + \frac{1}{w^2-1}\right]dw \quad \int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| +$$

$$= 2w\ln\left(w^2-1\right) - 4\left[w + \frac{1}{2}\ln\left|\frac{w-1}{w+1}\right|\right] + C$$

$$= 2w\ln\left(w^2-1\right) - 4w - 2\ln\left|\frac{w-1}{w+1}\right| + C$$

$$= 2x\sqrt{1+e^x} - 4\sqrt{1+e^x} - 2\ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C$$

$$= (2x-4)\sqrt{1+e^x} - 2\ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C$$

■

Solution to 57.

$$\begin{aligned}
w &= 1 + e^x \\
w - 1 &= e^x \\
\ln(w-1) &= \ln e^x = x \\
\frac{1}{w-1}\,dw &= dx \\
\int \frac{1}{1+e^x}\,dx &= \int \frac{1}{w}\,\frac{1}{w-1}\,dw \\
&= \int \frac{1}{w(w-1)}\,dw
\end{aligned}$$

$$\begin{aligned}
\frac{1}{w(w-1)} &= \frac{A}{w} + \frac{B}{w-1} \\
1 &= A(w-1) + Bw \\
\text{if } w = 0 : 1 &= -A \Rightarrow A = -1 \\
\text{if } w = 1 : 1 &= B \Rightarrow B = 1
\end{aligned}$$

Multiply both sides by $w(w-1)$.

Hence $A = -1, B = 1$. Then

$$\begin{aligned}
\int \frac{1}{1+e^x}\,dx &= \int \frac{1}{w}\,\frac{1}{w-1}\,dw \\
&= \int \frac{1}{w(w-1)}\,dw \\
&= \int \left[\frac{-1}{w} + \frac{1}{w-1}\right]\,dw \\
&= \int \left[\frac{1}{w-1} - \frac{1}{w}\right]\,dw \\
&= \ln|w-1| - \ln|w| + C \\
&= \ln\left|\frac{w-1}{w}\right| + C \qquad \text{use } w = 1+e^x,\ e^x, 1+e^x > 0 \\
&= \ln\left(\frac{e^x}{1+e^x}\right) + C
\end{aligned}$$

■

Solution to 58.

$$\begin{aligned} w &= e^x \\ dw &= e^x\,dx \\ \int \frac{e^x}{2e^{2x}+e^x+1}\,dx &= \int \frac{1}{2(e^x)^2+e^x+1}\,e^x\,dx \\ &= \int \frac{1}{2w^2+w+1}\,dw \\ &= \int \frac{1}{(2w-1)(w+1)}\,dw \end{aligned}$$

$$\frac{1}{(2w-1)(w+1)} = \frac{A}{2w-1} + \frac{B}{w+1}$$

Multiply both sides by $(2w-1)(w+1)$.

$$1 = A(w+1) + B(2w-1)$$

$$\text{if } w = -1 : 1 = -3B \Rightarrow B = \frac{-1}{3}$$

$$\text{if } w = \frac{1}{2} : 1 = \frac{3}{2}A \Rightarrow A = \frac{2}{3}$$

Hence $A = \frac{2}{3}, B = \frac{-1}{3}$. Then

$$\begin{aligned}
\int \frac{e^x}{2e^{2x} + e^x + 1}\, dx &= \int \frac{1}{(2w-1)(w+1)}\, dw \\
&= \int \left[\frac{\frac{2}{3}}{2w-1} + \frac{\frac{-1}{3}}{w+1} \right] dw \\
&= \frac{1}{3} \int \left[\frac{2}{2w-1} - \frac{1}{w+1} \right] dw \\
&= \frac{1}{3} \left[\ln|2w-1| - \ln|w+1| \right] + C \\
&= \frac{1}{3} \ln\left| \frac{2w-1}{w+1} \right| + C \qquad \text{use } w = e^x \\
&= \frac{1}{3} \ln\left| \frac{2e^x-1}{e^x+1} \right| + C = \ln \sqrt[3]{\left| \frac{2e^x-1}{e^x+1} \right|} + C
\end{aligned}$$

■

Solution to 59.

$$\begin{aligned} x^2 + 2x + 5 &= x^2 + 2x + 1^2 - 1^2 + 5, \\ &= (x^2 + 2x + 1) - 1 + 5, \\ &= (x + 1)^2 + 4. \end{aligned}$$

$$\int \frac{x}{\sqrt{x^2+2x+5}}\,dx = \int \frac{x}{\sqrt{(x+1)^2+4}}\,dx \qquad \text{note } ((x+1)^2+4)' = 2(x+1)$$

$$= \int \frac{x+1-1}{\sqrt{(x+1)^2+4}}\,dx \qquad \text{note } \int \frac{f'(x)}{\sqrt{[f(x)]^2+a^2}}\,dx = \ln\left|\frac{f(x)+\sqrt{[f(x)]^2+a^2}}{a}\right| + C$$

$$= \int \left[\frac{x+1}{\sqrt{(x+1)^2+4}} + \frac{-1}{\sqrt{(x+1)^2+4}}\right]dx \qquad \text{note } \int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C$$

$$= \int \frac{x+1}{\sqrt{(x+1)^2+4}}\,dx - \int \frac{1}{\sqrt{(x+1)^2+4}}\,dx$$

$$= \frac{1}{2}\int ((x+1)^2+4)^{-1/2}\,2(x+1)\,dx - \int \frac{1}{\sqrt{(x+1)^2+4}}\,dx$$

$$= \frac{1}{2}.2\sqrt{(x+1)^2+4} - \ln\left|\frac{x+1+\sqrt{(x+1)^2+4}}{2}\right| + C$$

$$= \sqrt{x^2+2x+5} - \ln\left|\frac{x+1+\sqrt{x^2+2x+5}}{2}\right| + C \qquad \text{note } x^2+2x+5 = (x+1)^2+4$$

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Solution to 60.

$$\begin{aligned}
w &= \sqrt{1+\sqrt{x}} \\
w^2 &= 1+\sqrt{x} \\
w^2-1 &= \sqrt{x} \\
(w^2-1)^2 &= x \\
2(w^2-1)(2w)\,dw &= dx \\
4w(w^2-1)\,dw &= dx
\end{aligned}$$

$$\int \frac{\sqrt{1+\sqrt{x}}}{x}\,dx = \int \frac{w}{(w^2-1)^2}\,4w(w^2-1)\,dw$$

$$= \int \frac{4w^2}{w^2-1}\,dw$$

$$= 4\int \left[\frac{w^2-1+1}{w^2-1}\right]dw$$

$$= 4\int \left[\frac{w^2-1}{w^2-1}+\frac{1}{w^2-1}\right]dw$$

$$= 4\int \left[1+\frac{1}{w^2-1}\right]dw \qquad \text{note } \int \frac{f'(x)}{[f(x)]^2-a^2}\,dx = \frac{1}{2a}\ln\left|\frac{f(x)-a}{f(x)+a}\right| + C$$

$$= 4\left[w+\frac{1}{2}\ln\left(\frac{w-1}{w+1}\right)\right]+C \qquad \text{use } w=\sqrt{1+\sqrt{x}}$$

$$= 4w + 2\ln\left(\frac{w-1}{w+1}\right)+C$$

$$= 4\sqrt{1+\sqrt{x}} + 2\ln\left(\frac{\sqrt{1+\sqrt{x}}-1}{\sqrt{1+\sqrt{x}}+1}\right)+C$$

■